A COMPARATIVE STUDY OF COMBINED GARCH & IV MODELS AGAINST INDEPENDENT GARCH & IV MODELS

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By: Group 26

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Abstract

Numerous models have been developed for forecasting volatility, some of the most popular ones being Implied Volatility (IV) and GARCH which work best in highly volatile and less volatile markets respectively. Therefore, this paper introduces a novel dynamic model which combines IV and GARCH into a single model that combines the strengths of both the models to provide all-round superior forecasting. This study compares the performance of individual IV and GARCH models against the Combined IV and GARCH(1,1) model on one month At-The-Money NIFTY50 options contracts over the past 10 years and finds that the combined model outperforms the individual models. This model can be used along with delta neutral options trading strategies to buy (sell) undervalued (overvalued) strategies by comparing the prices derived from this model against the market prices.

Key Words: *Volatility, Implied Volatility, GARCH*

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Introduction

Quantitative analysis of the financial markets is a method with the main focus on statistical analysis as a means to determine the value of a financial asset, for example, stocks or options. It uses a variety of data, including but not limited to, historical and real time prices for the purpose of creating trading algorithms and computer models. The end goal of quantitative analysis is to use quantifiable data to ensure that the investor makes an informed decision based on fundamentals. While both risk and returns are measured using statistical measures, a greater emphasis on risk is placed because while returns cannot be predicted accurately, risk can at least be managed. Some measures of risk are standard deviation, beta, Value-at-Risk (VaR), Conditional Value-at-Risk (CVar).

One of such statistical measures is Volatility, which is a measure of the dispersion of returns for a given market index or stock. Generally, the following is considered, the higher the volatility, the riskier the security. Volatility is many measured as either the standard deviation or variance between returns from that same financial instrument or market index. In the security markets, whenever there are major swings in either direction it is considered to be volatile. For example, when the stock market rises and falls over one percent over a substantial period of time, it is called a "volatile" market. One of the key factors when pricing an option contract is the asset's volatility.

Implied volatility (IV) is one such metric that is used to predict future moves and supply and demand, and is mostly used to price options contracts, high implied volatility results in options with higher premiums and vice versa. IV does not project the direction in which the price change will move. For example, high volatility would mean a significant price swing, but the price could swing upward—very high—downward—very low—or oscillate between the two directions. Low volatility means that the price change likely won't make large, unpredictable changes.

Autoregressive Conditional Heteroskedasticity (ARCH) is another such sophisticated and widely used model that is used to predict future volatility. Due to its superior performance, it has been iterated upon and a number of models have been derived off it such as GARCH, EGARCH, STARCH, TARCH to form a family of ARCH models. The most commonly used model out of the family of ARCH models is GARCH (Generalised Autoregressive Heteroskedasticity) due to its overall accuracy in general market situations when the markets are stable.

Important core tenets of the financial theory, Modern Portfolio Theory (MPT) and Efficient Market Hypothesis, assume that all participants in a financial market are rational, profit maximizing, and risk averse at all times. However, clusters of excess volatility are often observed when financial time series data is analysed, indicating a violation of the EMH since it means that prices deviate from fundamentals. Therefore, multiple studies have been conducted to explain why excess volatility exists and how can it be incorporated in models which has led to the widespread use of such econometric models by participants of financial markets in their pricing models and trading strategies. Since returns cannot be predicted but risk can be managed, we are attempting to determine which model gives us a better estimates of future volatility.

Problem Statement

There are numerous methods of calculating volatility in financial markets and some methods tend to perform more favorably than others during different periods and market conditions such as it is widely observed that Implied Volatility (IV) is one of the best performing models when the markets are highly volatile while GARCH provides best estimates when the markets are less volatile. However, IV performs poorly when the markets are stable whereas GARCH performs poorly when the markets are volatile in terms of forecasting error. Therefore, we wanted to determine whether a combined model of the two can be created to help cover each other's deficiencies since IV performs well in volatile markets while GARCH performs poorly in volatile markets but IV performs poorly in stable markets when GARCH performs better.

A single model that combines the strengths of both the models would help eliminate the need for finance professionals to keep switching both the models during different market conditions by providing an all-weather model instead.

Objective

The objective of this study is to prove that the forecasting error for combined IV and GARCH based models is greater than or equal to forecasting error of individual IV and GARCH models. This is achieved through making a new combined model of IV and GARCH.

Literature Review

Poon & Granger (2005) surveys 93 independently conducted studies employing different volatility models and concludes that the implied volatility of options provides a more accurate forecast than other time series models of volatility. The models surveyed in the time series category include historical volatility models, generalized autoregressive conditional volatility models and models based on stochastic volatility. The survey proves to be a practical guide to volatility modelling by highlighting commonly occurring issues in volatility models.

The work of **Kambouroudis, McMillan, & Tsakou** (2016) presents the case for using a model that that combines an asymmetric GARCH model with implied and realized volatility through (asymmetric) ARMA models. The success of this idea in US and European indices forms a solid motivation for the application of IV-GARCH models in the India Capital Markets. This study experiments with as many as 10 variants of the GARCH model to arrive at the best combination of IV, RV (Realized Volatility) and GARCH that minimizes a certain cost function. The best combination varies across different indices. This finding mandates the need for a researcher to choose an appropriate variant of the model that works best for a given index and cost function.

The work of **Mala & Reddy (2007)** on the stock market indices of Fiji Islands, brings up the role of interest rates in models that forecast volatility. This study finds a significant relationship between the volatility forecasts and interest rates. This relationship was established for Fiji Islands in the period 2001-2005. Emerging markets tend to be dependent on developed markets for capital. Changes in interest rates denominated in the major currencies of the world could possibly explain volatility of asset returns in emerging markets. If such a relationship can be brought to light in the Indian markets, the forecasting power of volatility models could be improved by introducing a variable representing such interest rates.

The study of **Christiansen, Schmeling, & Schrimpf (2012)** provided an in-depth analysis of volatility in the US financial markets with the help of economic variables. Some of the best predictors in this model were those which had a sensible economic interpretation. For example, Valuation ratios for equities, Interest rate differentials in foreign exchange and variables that proxy for credit risk and funding liquidity. In contrast, variables that proxy for macroeconomic conditions are much less informative at predicting future volatility. Variables that emphasized the effects of leverage, credit risk, funding illiquidity and time-variation of risk premia were the best ones to understand the relation between economics and volatility.

Volatility Estimation plays a vital role when it comes to various areas of finance such as derivatives pricing, VaR calculations and other price forecasting models as well. In their paper, **Rastogi, Don, & V (2018)** have explored the family of GARCH models to explore volatility in the Indian markets. A comparative analytical approach between the different models in GARCH using one sample t-test showed that I-GARCH was the only model which statistically had similar results to implied volatility in the options market. This proved that the properties which are accounted in the I-GARCH model were the most likely to explain the properties of volatility in the financial markets. This conclusion provided a prove that past information is likely to be seen in the future, violating the theories of efficient market hypothesis and mean reversion.

The research of **Donaldson & Kamstra (2004)** proved that volume had a powerful role at forecasting volatility, with volume playing a vital role of a switching variable between states in which IV was more informative than ARCH for volatility predictions. The research revealed that if volume was lower than normal, then the best forecast for volatility was by giving equal weights to ARCH and IV whereas if it was lower than normal, then the best forecast was by giving more weights to IV as compared to ARCH.

The study by **Degiannakis (2004)** tries to bring out a higher degree of accuracy for predicting the one-step-ahead volatility of stock returns by extending the ARCH model to further capture the skewness and excess kurtosis of the return distribution and also the fractional integration of conditional variance, this helps the model to capture the different effects of positive and negative errors in prediction. Although this research does conclude that the extended model of ARCH i.e., FIAPARCH is significantly more accurate in predicting the one-step-ahead volatility of stock returns, we do not plan to use this extended model in our research since the Implied volatility model gives very similar insights for prediction.

Baroian (2014) examines whether the instability of macroeconomic variables affect the stock market volatility. A panel of 5 European countries were taken in the study where modified ARCH/GARCH models were used to account for the effects of fundamental factors. The article concluded that exchange rate volatility was the only explanatory variable, while initially the relationship in case of the 5 countries was not constant, after removing the distinct features that were being captured in the fixed effects, the relationship between exchange rate volatility and the securities market volatility was established to be positive.

The work done by **Engle & Patton (2001)** focuses on the various factors of volatility that should be incorporated in a volatility model. Some of the factors being persistence, mean reversion, exogenous variables, etc. Twelve years of daily data was collected from the Dow Jones Industrial index to exhibit the ability of GARCH, ARCH-type models to inculcate these factors. the log-difference of the value of the index, so as to transform the data into continuously compounded returns. They were able to figure a few short-comings of above-mentioned models, the most prominent one being, the hypothetical perception that in the event that a GARCH model is accurately indicated for one recurrence of information, it will be mis specified for information with various time scales.

Hansen & Lunde (2005) did a comparison of 330 ARCH-type models on their capability to describe the conditional variance. The models were compared out-of-sample using six different loss functions. Daily DM–$ exchange rate data and daily IBM returns was used as data for the purpose of this study. The study revealed that the benchmark model, ARCH (1) or GARCH (1,1), was the best model when comparing exchange rate data. Though in the analysis of IBM stock returns we found conclusive evidence that the GARCH (1,1) is inferior to those models which could accommodate a leverage effect.

The study done by **Pandey (2005)** stated that there were four possible approaches for estimating and forecasting volatility. Traditional volatility and Extreme value volatility estimators, conditional volatility model and implied volatility. This study compares the outcome of the first three approaches in estimating and forecasting Nifty returns. The data that was used was high frequency data on S&P CNX Nifty, a value-weighted stock index of National Stock Exchange (NSE). The analysis was done on the performance of the volatility models and estimators vis-à-vis the realized volatility of the S&P CNX Nifty stock index in the terms of estimation and predictive power. The results of the study indicated that even though conditional volatility models perform well in estimating volatility for the past in terms of bias, extreme value estimators based on observed trading range perform well on efficiency criteria. For forecasting purposes, the extreme value estimators were able to forecast five-day (approximately a week) and one-month volatility ahead — much better than conditional volatility models.

In their article **Byun & Cho (2013)** examined the volatility forecasting abilities of three approaches namely GARCH-type model that uses carbon futures prices, an implied volatility from carbon options prices, and the k-nearest neighbour model to examine the predictive power of GARCH-type, IV, and k-NN models of EUA futures returns and to verify if the EUA futures volatility is correlated to energy market volatility. According to the results, GARCH-type models turned out to be superior than an implied volatility and the k-nearest neighbour model probably due to the low trading volume of carbon options. Upon investigating the volatilities of energy markets using linear regression analysis, it was found that Brent oil, coal, and electricity may be used to forecast the volatility of carbon futures. The results of the superlative models enabled market participants to better hedge their positions by improving the forecasting volatility of carbon futures.

The research conducted by **Rapach & Strauss (2008)** focused upon the empirical relevance of structural breaks for GARCH models of exchange rate volatility using in-sample as well as out-of-sample tests. For in-sample analysis, an upgraded version of Inclan and Tiao (1994) iterated cumulative sum of squares algorithm that allows for dependent processes was employed. The algorithm was applied to test for structural breaks in the unconditional variance of everyday US dollar exchange rate returns with regard to the currencies of seven OECD countries, as well as day-to-day returns for a trade-weighted US dollar exchange rate. For the out-sample analysis, various methods were taken into consideration for making it complaint for potential structural breaks when forming exchange rate return volatility forecasts in real time. Daily nominal exchange rate data from Global Financial Data to compute the daily return of the US dollar against the currencies of Canada, Denmark, Germany, Japan, Norway, Switzerland, and the UK for January 2 1980 to August 31 2005 was used for the purpose of the study. The study revealed that structural breaks are an empirically relevant phenomenon for GARCH models of US dollar exchange rate return volatility.

In their research work **Goudarzi & Ramanarayanan (2011)** the authors studied the effects of various market sentiments on volatility in the Indian stock markets using asymmetric ARCH models during the global financial crisis of 2008-09. Volatility was modelled using two specified nonlinear asymmetric models, EGARCH (1, 1) and TGARCH (1, 1) and news impact curve. The BSE500 stock index was used as a proxy to the Indian stock market to study the asymmetric volatility over 10 years' period. It was found that BSE500 returns series exhibit leverage effects implying that the negative innovation (news) has a greater impact on volatility than a positive innovation (news). Moreover, they exhibit other stylized facts such as volatility clustering and leptokurtosis associated with stock returns on developed stock markets indicating significant influence of the sign of innovation on the volatility of returns and the arrival of bad news in the market. Therefore, proving bad news in the Indian stock market influenced volatility more than good news.

Pilbeam & Langeland (2015) compared the forecasts derived from the IV and three different univariate GARCH models in the foreign exchange markets at pricing option volatility by dividing the data into two periods, namely 2002 to 2007 which is characterized by low volatility and 2008 to 2012 which is characterized by high volatility. In the first period, the IV performs well as a predictor, however, in the high volatility period the implied volatility performed poorly in predicting actual volatility. But overall, it was found that the implied volatility forecasts outperform the three GARCH models in both low and high volatility periods therefore suggesting that the foreign exchange market efficiently prices foreign currency options, thus suggesting a deficiency of the three univariate GARCH models used in forecasting the volatility of the foreign exchange market.

Research Methodology

Hypothesis

Null Hypothesis (H_0)

Forecasting error for combined IV and GARCH based models is greater than or equal to forecasting error of individual IV and GARCH models

Alternate Hypothesis (H_1)

Forecasting error for combined IV and GARCH based models is lesser than forecasting error of individual IV and GARCH models

Data Collection and Processing

Collecting Data and bringing it to usable form is a three-step process:

1. ***Collecting prices of one month ATM NIFTY options and the underlying Nifty:*** The raw data required for this study is the prices of one month ATM NIFTY options and the price of Nifty in the appropriate period. The historical prices of Nifty can be easily extracted from the old website of NSE. Collecting the prices of ATM options is tricky because the strike price of the ATM option will change as the underlying Nifty changes. These prices are therefore extracted algorithmically, by re-calculating the strike price of the ATM option daily (on the basis of the closing price of underlying).
2. ***Estimating Implied Volatility:*** Using the price of the option, price of the underlying, the risk-free interest rate on that date (annualized yield on the 10-year G-sec) and the time to expiry of that particular option, we have estimated the implied volatility of the options on a daily basis.
3. ***Estimation of GARCH forecast:*** The GARCH forecast is made directly on the underlying. The GARCH model is re-estimated every day on a rolling basis, taking into account the previous 123 observations of the underlying.

Models

The model used in this study- IV, GARCH and our newly proposed combined IV and GARCH models are explained below.

Implied Volatility (IV)

Implied Volatility (IV) estimates the future volatility in the underlying stock which is calculated based on option prices. It can act as a precious tool for option traders in order to determine if the option prices are cheap or expensive. Since, option prices are determined based on demand and supply of the contract, this measure helps us figure out how much is the expected volatility in the market according to the participants in the derivatives market.

Implied volatility also has an effect on the pricing of non-option financial instruments, for example: an interest rate cap, which put a limit on the amount an interest rate on a product that can be raised. Implied volatility can be calculated though the use of an option valuation model, one example of such a model is the Black-Scholes Model. It is one the most widely used model, which factors in current stock price, option strike price, time until expiration, and risk-free interest rates. IV, just like the markets as a whole, is subject to unpredictable changes. Supply and demand are few of the major factors in determining IV. While IV helps in quantifying market sentiment, set option prices, and determining trading strategy, it does have a few short-comings. It is based solely on price and not fundamentals, extremely sensitive to unexpected factors, and even though it may predict movement, it does not predict the direction.

When it comes to the calculation of Implied Volatility in the market, it is done by inputting all the information we have in Black-Scholes Model of options valuation and back-calculate the equation for volatility. The equation for Black-Scholes is as follows, For Call Options,

$$C = N(d_1)S_t - N(d_2)$$

For Put Options,

$$P = Ke^{-rt}N(-d_2) - S_tN(-d_1)$$

Where,

$$\mathrm{d}_1 = \frac{Ln\left(\frac{S_t}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}$$

$$\mathrm{d}_2 = d_1 - \sigma\sqrt{t}$$

Where,

C = Call Option price

P = Put Option Price

S_t = Spot Price of Underlying

K = Strike Price

r = Risk-free interest rate

t = Time to Maturity

σ = Standard Deviation of an asset

For the purpose of our research, we have referred to monthly European options of NIFTY50 to calculate the IV of our model. At-the-money strike prices have been selected based on the closing price of the day in order to calculate the day’s implied volatility.

GARCH

Generalized Autoregressive Conditional Heteroskedasticity (GARCH) is a conditional variance model that is used to forecast the volatility of financial time series. It was developed by the economist Dr. Tim Bollersev (Bollerslev, 1986), to study an approach to estimate volatility in financial markets by allowing a longer memory and a more flexible lag structure than the ARCH model. Since its initial development in 1986, it has been iterated upon to create different variations such as Exponential GARCH (EGARCH), Nonlinear GARCH (NGARCH) and Integrated GARCH (IGARCH) which address conditional heteroskedasticity, correlations, and non-stationarity of covariance respectively.

The symmetric model we use is the GARCH (1,1) model of the GARCH generalisation proposed in Bollerslev (1986) in which volatility at time *t* is also affected by *p* lags of past estimated volatility. It returns weighted average of past squared residuals but it has declining weights (Pilbeam & Langeland, 2015), i.e. it provides a more real world scenario than other forms when trying to predict the volatility of financial instruments which is extremely useful with risk management, asset allocation, portfolio optimization and hedging decisions. The specification of a GARCH (*p*,*q*) is given by:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \, \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \, \sigma_{t-j}^2$$

Where,

ω, α_1, …., α_q, β_1, …, β_p are the parameters to be estimated

q is the number of return innovation legs included in the model

p is the number of past volatility lags included in the model

Proposed Model (Combined IV and GARCH model)

This research proposes a new model that captures the asymmetrical relationship between market returns and implied volatility. The implied volatility of options tends to be higher when markets are falling as compared to when markets are rising. An equitable market movement in the upward and downward direction will impact implied volatility differently. The proposed model seeks to capture this asymmetrical impact of market returns to predict the market implied volatility for the next day.

In essence, the proposed model forecasts future implied volatility as a weighted average of present implied volatility and the forecast derived from a GARCH(1,1) model on the underlying. This study uses the Nifty 50 as a representative of the market. Market implied volatility is the implied volatility of the At-the-Money (ATM) European call option on the nifty.

Mathematically, the model is defined as follows:

$$\sigma_{t+1} = \phi_1 * IV_t \ + \ \phi_2 * \sigma'_{t+1} \ldots \ldots (1)$$

$$\phi_1 \ + \ \phi_2 = 1 \ldots \ldots (2)$$

$$\phi_1 = \frac{e^{\delta}}{(1 + e^{\delta})} \ldots \ldots (3)$$

$$\delta = g(r_t) = \begin{cases} -(r_t)^{(\frac{1}{p})}, & r_t < 0 \\ r_t, & r_t \geq 0 \end{cases} \ldots \ldots (4)$$

$$-1 < r_t < 1 \ldots \ldots (5)$$

Where $\sigma_{t+1}$is the predicted implied volatility of the ATM call option on the Nifty with the most recent monthly expiry. ϕ_1 is the weight given to the present implied volatility on the described option. ϕ_2 is the weight given to the GARCH (1,1) forecast on the underlying nifty for the next trading day and $\sigma'_{t+1}$is the GARCH described GARCH forecast. We use (3) and (4) to compute the weight of IV and use (2) to compute the weight of the GARCH forecast. The p in (4) is a hyper parameter that control the asymmetry discussed above. p has to be greater than or equal to 1. The greater its value, the greater is the extra weight given to market implied volatility during falling markets. In this paper, we have pseudo-optimized the value of p using a trial-and-error approach. This hyper parameter can however be optimized using appropriate optimizing algorithms on defined cost functions. A reasonable way of doing that is by minimizing (using the restrictions on p as constraints) the squared error of forecast from (1) and the actual IV of option by the end of the next trading day. r_t is the log return of the market at time *'t'*. It has been mathematically bounded between -1 and 1 to reflect that the function $g(r_t)$ is defined only for this range of r_t. In practice, regulators and exchanges have a hard circuit of 20% (0.2). If markets move more than this in a day, trading is immediately suspended. Therefore, the restriction on the domain of $g(r_t)$ is reasonable.

Data Analysis

We have implemented the earlier described model for a period of over 10 years in the context of the Nifty 50 index. The GARCH model is estimated on a rolling basis (re-estimated as many times as there are one day ahead forecast). For the purposes of IV, we have computed the IV of the options based on the daily closing price of the ATM option. The definition of ATM changes every few days with changes in the underlying index. In this study we have adjusted for such changes. The other variables used as the time to expiry of the option and interest rate (yield on the 10-year G-sec of the Government of India). We compute the forecasts generated by the proposed model and compare them to the actual implied volatility on the following day. We take a squared difference of our forecast and that of the actual IV. We repeat the same process for the GARCH forecast and a pure IV based forecast (where todays realized IV becomes the forecast for tomorrow’s IV) to facilitate comparison. We then individually compare the squared error of the proposed to model to both the comparison models. We conduct an f-test to determine whether the error of the proposed model is significantly lower than that of the two comparison models. The table below highlights the results of the f-test.

H0: The ratio of squared error of a pure IV forecast and squared error of our model is 1

H1: The said ratio is greater than 1

Model	Average Error	Ratio	p-value
IV+GARCH	0.00104%	1.1524	1.26E-04
IV	0.00119%		

Table 1: Comparison between Proposed Model and Pure IV Model

H0: The ratio of squared error of a pure GARCH forecast and squared error of our model is 1

H1: The said ratio is greater than 1

Model	Average Error	Ratio	p-value
IV+GARCH	0.00104%	2.0169	3.26E-72
GARCH	0.00209%		

Table 2: Comparison between Proposed Model and Pure GARCH Model

As it is seen above, the proposed model has significantly (at 99% significance) outperformed both the comparison models. The error of the IV model is 1.1524 times the error of the proposed model. Similarly, the error of the GARCH model is 2.0169 times the error of the proposed model. Earlier research indicates that IV tends to overestimate future volatility in the underlying. Government and corporate activities and rapid changes in economic and geopolitical situations cause market panic. GARCH is not capable of detecting these panic situations as it bases its estimate on past realized volatility. In a broad sense, the proposed model dynamically switches between IV and GARCH estimates based on market conditions. As a result of this, its performance is superior to both the models individually. To better analyze the proposed model, we estimate three regression equations that employ the proposed model and two comparison models respectively in order to show which model's forecast is more closely related to the actual future volatility. Although the f-test provides similar insights, we continue with the regression estimations because traditionally IV has been used as an independent variable in a regression to forecast volatility. The results of the same are displayed below.

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.730238419
R Square	0.533248149
Adjusted R Square	0.533073007
Standard Error	0.003215145
Observations	2667

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.031473254	0.03147	3044.67	0
Residual	2665	0.027548524	1E-05		
Total	2666	0.059021778			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.002830715	0.000152181	18.601	1E-72	0.002532311	0.00312912	0.002532311	0.00312912
Lagged IV	0.730212538	0.013233631	55.1785	0	0.704263313	0.756161764	0.704263313	0.756161764

Table 3: Regression estimate for pure IV model

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.304553149
R Square	0.092752621
Adjusted R Square	0.09241219
Standard Error	0.004482501
Observations	2667

ANOVA

	df	SS	MS	F	Significance F
Regression	1	0.005474425	0.005474425	272.4568178	2.3623E-58
Residual	2665	0.053547353	2.00928E-05		
Total	2666	0.059021778			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%	Lower 95.0%	Upper 95.0%
Intercept	0.003938733	0.000406454	9.690478968	7.55634E-22	0.003141736	0.00473573	0.003141736	0.00473573
Garch	0.615934514	0.037315194	16.50626602	2.3623E-58	0.542764847	0.689104181	0.542764847	0.689104181

Table 4: Regression estimate for pure GARCH model

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.73101781
R Square	0.534387039
Adjusted R Square	0.534212324
Standard Error	0.00321122
Observations	2667

ANOVA

	df	SS	MS	F	Significance F
Regression	1	0.031540473	0.031540473	3058.637915	0
Residual	2665	0.027481305	1.03119E-05		
Total	2666	0.059021778			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%	Lower 95.0%	Upper 95.0%
Intercept	-0.000812612	0.000213671	-3.803094847	0.000146107	-0.00123159	-0.000393634	-0.00123159	-0.000393634
Forecast	1.067106259	0.019294949	55.3049538	0	1.02927167	1.104940847	1.02927167	1.104940847

Table 5: Regression estimate of proposed model

The three regression estimates further validate the proposed model. The f-statistic for the proposed model is 3058.63, while that of the pure GARCH and pure IV models is 272.46 and 3044.67 respectively. Furthermore, the t-stat of the coefficients also prove that the proposed model is most significant. Similar is the case of the Adjusted R Square. The three regression estimates prove that the proposed model is explaining the actual future IV much better than the other two comparison models.

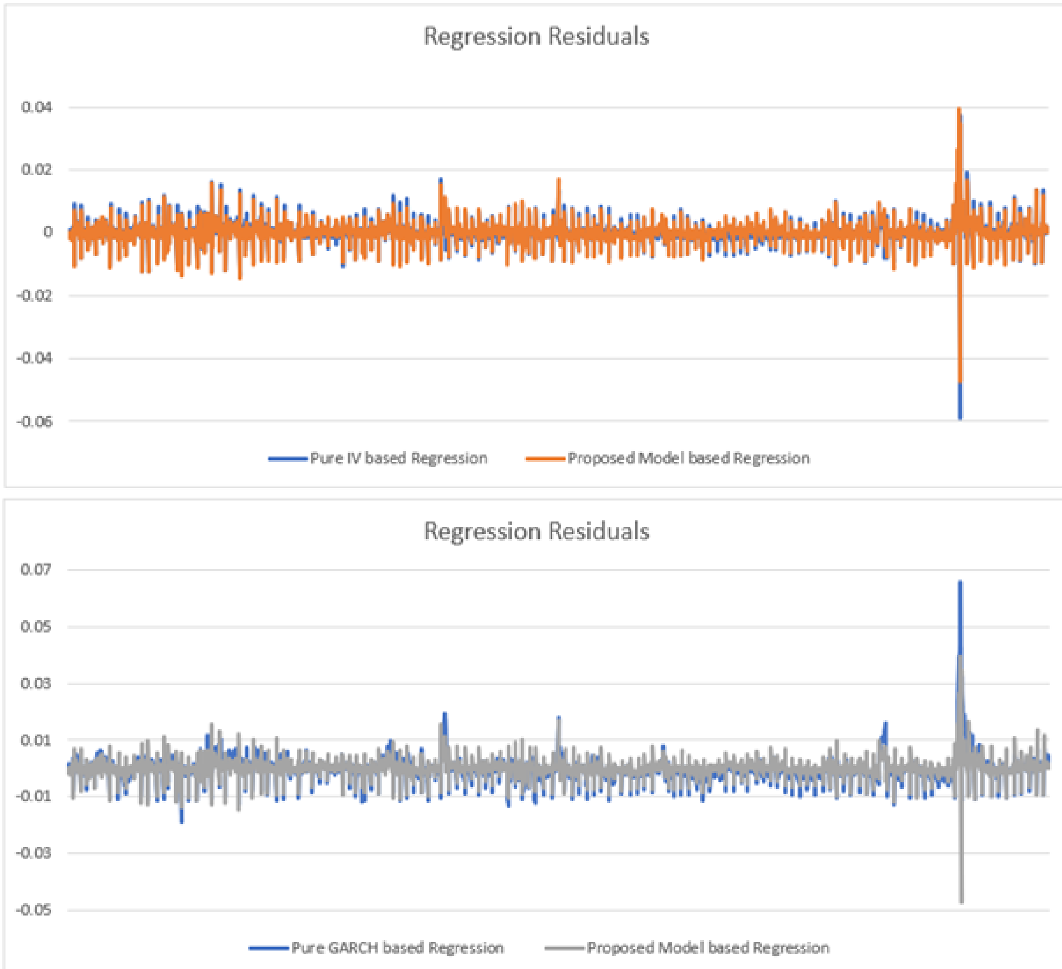

Figure 1: Comparison of Residuals

An analysis of the residuals of the regression conducted using the model forecast and pure IV forecast as the independent variables. The figure above plots the residuals of both the regressions above. The residuals of the regression based on forecasts of proposed model are represented in orange. The figure shows the blue lines extending beyond the orange lines. The blue line is showing greater variation than the orange line. This is visual proof that the regression based on pure IV is making more error than the regression based on the forecast of proposed model. Similar is the case when we compare the regression based on pure GARCH forecasts with the regression based on proposed model forecasts. In the figure below, the blue line is observed to be more volatile than the grey line. Even during the big shock towards the right side of the figure, the size of the downward shock in the grey line is much lower than the size of the upward shock in the blue line.

Conclusion

Decision making in finance is dependent upon the tradeoff between risk and return, making empirical statistical analysis and measurement of risk is an integral part of risk management, option pricing and portfolio management. Therefore, numerous studies that investigate the performance of various models used for estimating the volatility of financial markets have been conducted. We have combined two such well-known and well performing models, namely IV and GARCH to create a dynamic model which combines the complementary strengths of both the individual models.

As demonstrated, the newly proposed Combined IV and GARCH model performs better than the independent IV and GARCH models at measuring volatility of monthly at-the-money NIFTY50 option contracts in overall market scenarios due to its ability to dynamically move towards independent IV and GARCH measures depending upon the relevant favourable market conditions i.e., our model gives higher weightage to IV when the market conditions are highly volatile and gives a higher weightage to GARCH when the market conditions are less volatile. The combined model can then be used to determine when the IV estimation is overvalued or undervalued, and accordingly take up positions and apply relevant trading strategies on NIFTY50 Options Contract such as buying the contract when the IV is undervalued and selling the contract when the IV is overvalued as compared to our combined model.

One application of our proposed model is combining it with Delta-Neutral strategies by calculating a volatility estimate using our model and then using it in the Black-Scholes Merton formula to forecast the closest ATM straddle price. If the forecasted Straddle or Strangle Price is greater than the market price of the strategy, it will be bought and if the forecasted price is less than the market price, it shall be sold. Other than Delta Neutral spreads this model can also be used in determining if we want to buy or sell options in Directional Strategies.

Limitations

While calculating the weights for GARCH and IV in our model, we are assuming a deterministic return where we are taking the closing price of the previous day in order to calculate daily returns. In reality, prices can be highly volatile where your low prices can give returns as far as –5% intraday but the actual return at the end of the day might just be –1% which will lower our weights to IV but does not accurately capture panic for the entire trading day.

Since IV is calculated in the Black-Scholes model by inputting all the other factors, sometimes the calculation for IV can be a lot different in our model as compared to others. For example, NSE option chain assumes a constant interest rate of 10% while computing the IV whereas our model has a varying interest rate based on 10 Year G-Sec yields. Also, IV is calculated by testing different levels of volatility to reach at our actual prices. To do this, some algorithms can be more powerful as compared to others and this might bring a variation in IV values.

In this model, we have considered the normal GARCH (1,1) as one of our models and have not compared it with other models in the ARCH family for a more comprehensive evaluation.

Also, since $g(r_t)$ in equation (4) of our GARCH model is not differentiable at $r_t=0$ we have no solid mathematical basis of optimizing the hyper parameter P. This motivates us to do more research on asymmetrical functions that are differentiable throughout their domains.

The entire process of calculating volatility through our model is very heavy computationally. This can act as a huge drawback while trading in real time since we are calculating using both IV and GARCH.

Another limitation to IV is the fact that we have considered that ATM options are delta neutral in nature. A more practical approach to this would be considering the combined delta of the options at the same strike to consider delta neutral strikes.

When we are talking about the application of our model in delta neutral spreads, we plan on taking advantage of our view on volatility by taking spreads which has a delta of 0. Ultimately, even after neutralizing our delta we have a very high gamma exposure which is not taken care of. If the volatility is decreasing but the prices are moving in one direction, it can cause a spike in delta values and break our trades even after being right.

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